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Competition and Bank Risk Taking in a Differentiated Oligopoly

Importante

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Abstract

We re-examine the relationship between the degree of deposit market competition and bank risk taking in a model where banks compete in differentiated deposit services. When banks invest their deposits directly, as has already been established in the extant literature, an increased degree of competition, measured either by greater degree of substitutability or by greater number of banks, induces the banks to take more risk in equilibrium. When banks invest their deposits in loans, and their borrowers choose the level of risk, the risk of bank failure is independent of the degree of competition in the deposit market.

Keywords: Bank competition; risk taking; loan contracts.

Resumen

Examinamos la relación entre el grado de competencia en el mercado de depósitos y el riesgo bancario en un modelo en el que los bancos compiten en servicios diferenciados. Cuando los bancos invierten directamente sus depósitos, como ya se ha establecido en la literatura existente, un mayor grado de competencia, medido ya sea por un mayor grado de sustitutibilidad o por un mayor número de bancos, induce a los bancos a tomar más riesgo en equilibrio. Cuando los bancos invierten sus depósitos en préstamos, y sus prestatarios eligen el nivel de riesgo, el riesgo de quiebra es independiente del grado de competencia en el mercado de depósitos.

Palabras clave: Competición bancaria; toma de riesgo; contratos de préstamo.
Introduction

It is well-known that when banks are able to invest their deposits directly in risky projects, increased degree of deposit market competition induces banks to take more risk (e.g. Matutes and Vives, 1996, 2000; Repullo, 2004). A lower intermediation margin, implied by more competition in deposit market, incentivizes banks to take more risk as riskier projects, if successful, yield higher returns. This is due to the implied moral hazard problem as in case a project fails the bank is not required to repay its depositors since it is protected by limited liability.¹ Boyd and De Nicoló (2005), in their seminal paper, show that when banks are able to invest their deposits only in loans, the so-called positive relationship between competition and risk is reversed. In particular, they consider a banking sector where financial intermediaries compete in homogeneous deposit and loan services. Since in a lending relationship the borrowers face the moral hazard problems in choice of risk, incentive compatibility implies a positive association between the level of risk and loan rate. Therefore, greater competition induces lower rates in equilibrium, and diminishes the risk of bank failure, which is the risk-shifting effect of competition.

In the present paper we show that the negative relationship between competition and risk as established in Boyd and De Nicoló (2005) is possible only if there exists externality among the lending relationships. In particular, we show that if the project returns are independent across borrowers, then the equilibrium levels of risk and loan rates do not depend on the characteristics of the deposit market. We analyze a simple and tractable model of banking sector where banks compete in differentiated deposit services. Thus, both the degree of substitutability and the number of banks measure the degree of deposit market competition. When banks can directly invest their deposits, we find the usual positive relationship between competition and risk taking – the greater the degree of substitutability or the greater the number of banks, higher is the risk taking in equilibrium. When banks can invest their deposits only in loans, independence of project returns across borrowers implies that equilibrium loan rates and levels of risk are determined entirely by individual project characteristics, and do not depend on the characteristics of the deposit market.

The main driving force behind the result of Boyd and De Nicoló (2005, Proposition 2) is that the loan contracts are subject to externality induced by a common demand function for homogeneous loans faced by the banks. Their result crucially depends on how the levels of competition in the deposit and loan markets are measured. Boyd and De Nicoló (2005) measure competition in both markets by the number of banks in the economy. If the competition is in differentiated deposit and loan services, the degrees of substitutability of deposits and loans are also appropriate measures of competition in the respective markets, and the degree of substitutability in one market is not necessarily the same as that in the other.² Dam et al. (2014) show that in such a context the equilibrium risk depends on the level of competition of the loan market only. Martínez-Miera and Repullo (2010) extends the model of Boyd and De Nicoló (2005) by introducing correlation in loan defaults. They identify two countervailing effects: in addition to the usual risk-shifting effect there is a margin effect which implies that lower loan rates caused by greater competition decrease banks' revenues from

1 In a dynamic model of competition it is often argued that low charter value due to increased competition is the main reason for banks to get involved in high-risk investments (e.g. Hellmann et al., 2000; Repullo, 2004).
2 Oligopolistic competition in differentiated banking products has been modeled earlier in the literature. Matutes and Vives (1996, 2000) have used a linear-city model à la Hotelling, whereas Chiappori et al. (1995); Repullo (2004); Dam et al. (2014) have analyzed banking competition in a circular-city model à la Salop (1979).
performing loans, which provide a buffer against loan losses. As a consequence, greater competition implies higher risk. Thus, there is in general a non-monotone relationship between competition and risk of bank failure. The main contribution of the present paper lies in providing a unified framework for deposit market competition where both the degree of substitutability and number of competitors measure the level of competition. Then, by introducing a loan market where the return of each project is subject only to idiosyncratic shocks, we show that the equilibrium loan contracts and risk levels neither depend on the degree of product substitutability nor on the number of competitors in the deposit market. Therefore, the behavior of equilibrium risk of bank failure with respect to competition crucially depends on the appropriate measure of competition and the distribution of project returns.

1 The model

Consider a banking sector consisting of two classes of risk neutral agents: a representative depositor and \( n \geq 2 \) banks. The economy lasts for 3 dates. At \( t = 0 \), banks simultaneously raise deposits by offering deposit rates \( r = (r_1, \ldots, r_n) \). The supply of deposits with bank \( i \) is thus given by \( D_i(r_i, r_{-i}) \), where \( r_{-i} = (r_1, \ldots, r_{i-1}, r_{i+1}, \ldots, r_n) \) with \( r_i > 1 \) for all \( i \). Deposit services offered by banks are differentiated with the degree of substitutability \( \delta \in (0, 1) \). The inverse supply function of deposits of bank \( i \) is given by:

\[
r_i = 1 + D_i + \delta D_{-i},
\]

where \( D_{-i} = \sum_{j \neq i} D_j \). Therefore, the supply of deposits with bank \( i \) is given by:

\[
D(r_i, r_{-i}) = \frac{-(1 - \delta) + [1 + \delta(n - 2)]r_i - \delta r_{-i}}{\Delta(\delta, n)},
\]

where \( \Delta(\delta, n) := (1 - \delta)[1 + \delta(n - 1)] \). The degree of deposit market competition is measured by two parameters: the degree of substitutability \( \delta \) and the number of banks \( n \).

At date 1, banks simultaneously choose their investment strategies. Each bank \( i \) has access to a continuum of constant-returns-to-scale risky technologies indexed by \( \theta_i \in [0, 1] \). In particular, given an investment \( I \), technology \( \theta_i \) yields:

\[
f(I, \theta_i) = \begin{cases} 
(\alpha + \theta_i)I & \text{with probability } 1 - \theta_i, \\
0 & \text{with probability } \theta_i,
\end{cases}
\]

with \( \alpha \in [1, 3] \). The assumption that \( \alpha \geq 1 \) implies that a safe project viable, whereas \( \alpha \leq 3 \) does not rule out a risky project to be undertaken. Note that higher values of \( \theta_i \) imply greater per unit return, but at the same time, higher probability of failure, and hence a riskier project.\(^4\) The choice of risk

\(^3\)The following supply function may be derived from a quadratic indirect utility function of the depositor which is given by:

\[
V = \sum_{i=1}^{n} r_i D_i - \left( \frac{\sum_{i=1}^{n} D_i}{2} + \delta \sum_{i=1}^{n} D_i + \delta \sum_{i=1}^{n} \sum_{j \neq i} D_i D_j \right).
\]

\(^4\)All our results hold good if we have assumed instead a decreasing-returns-to-scale technology \((\alpha + \theta_i)^{\gamma} \) with \( \gamma \in (0, 1) \), and a probability of success function \( \pi(\theta_i) \) where \( \pi'(\theta_i) < 0 \) and \( \pi''(\theta_i) \leq 0 \) for all \( \theta_i \in [0, \bar{\theta}] \) with \( \pi(0) = 1 \) and \( \pi(\bar{\theta}) = 0 \).
by each bank cannot be observed by the depositor, and hence banks face a potential moral hazard problem. Deposit contracts are subject to the banks' limited liability meaning that in the case the project of a bank $i$ fails it is not obliged to pay back the depositor. As a consequence, banks may opt for riskier project since it earns higher income with a positive probability. Such incentive problems of the banks will be crucial for the analysis of the deposit market competition where banks choose their own investment strategies. We assume, without loss of generality, that the deposits are fully insured by the central banking authority or by some insurer, e.g. the FDIC who charges a flat per unit insurance premium which is normalized to zero. Finally at $t = 3$, the project returns are realized and the depositor is paid off.

2 The deposit market equilibrium

We analyze a symmetric subgame perfect Nash equilibrium (SPNE) of the deposit market game in which all banks choose the same levels of risk $\theta$ and the same deposit rates $r$.\footnote{Since the banks are ex-ante identical, it is easy to show that there is a unique symmetric SPNE, and there are no asymmetric equilibria.} At $t = 1$, given the supply of deposits $D_i(r_i, r_{-i})$, each bank $i$ solves

$$\max_{\{r, \theta_i\}} (1 - \theta_i)(a + \theta_i - r)D_i(r_i, r)$$

subject to $\theta_i = \arg\max_{\theta_i} \left\{ (1 - \theta_i)(a + \theta_i - r)D_i(r_i, r) \right\} = \frac{1}{2}(1 - a + r) \equiv \theta(r_i).$ (ICB$_i$)

Constraint (ICB$_i$) is the incentive compatibility constraint of bank $i$ which asserts that the bank will choose the risk level in order to maximize its expected payoff from deposit services given the deposit rates announced by all banks at the previous stage. The following proposition analyzes the behavior of the equilibrium deposit rate and risk taking with respect to the degree of deposit market competition.

**Proposition 1** Let $r(\delta, n)$ and $\theta(\delta, n)$ be the deposit rate and risk shifting, respectively in the symmetric SPNE.

(a) For a given number of banks $n$, the equilibrium deposit rate and risk level are monotonically increasing in the degree of substitutability $\delta$.

(b) For a given degree of substitutability $\delta$, the equilibrium deposit rate and risk level are monotonically increasing in the number of banks $n$.

The proofs of the above proposition and the other subsequent results are relegated to the appendix. Recall that we measure the degree of competition either by the degree of substitutability or by the number of banks. When competition increases, in order to attract more deposits each bank offers a higher deposit rate. As the intermediation margin $m(\delta, n) := a + \theta(\delta, n) - r(\delta, n)$ of each bank is lower with greater degree of competition, banks tend to take more risk in the expectation of maintaining a positive margin. Note that the intermediation margin is monotonically decreasing in both $\delta$ and $n$ with $\lim_{\delta \to 1} m(\delta, n) = \lim_{n \to \infty} m(\delta, n) = 0$ as $\delta \to 1$ corresponds to perfect substitutability of
deposit services, and \( n \to \infty \) implies a perfectly competitive banking sector. The above results can be seen as a generalization of the results already established in the literature in the sense that risk taking in a banking sector is positively correlated with the degree of competition in the deposit market irrespective of how the degree of competition is measured.

3 Effect of loan contracts on risk taking

Boyd and De Nicoló (2005) show that when banks invest in loans, and their borrowers are the one who choose the riskiness of the projects, the well-known positive relationship between competition and risk taking is reversed. They consider an economy where banks compete in homogeneous deposit and loan services, and hence the number of banks \( n \) measures the degree of competition in both markets. They show that risk taking by banks decreases with the number of banks in the industry. In what follows we argue that the equilibrium level of risk in the banking sector is actually independent of the degree of competition in the deposit market, and hence there is no monotone relationship between competition in deposit market and equilibrium risk shifting.

In order to analyze the effect to loan contracts on risk taking, add two more dates to the initial timeline described in Section 1. After each bank \( i \) has its deposits \( D_i \) in date 1, in the following date bank \( i \) lends up to \( D_i \) at a loan rate \( \rho_i \) to a risk neutral borrower/entrepreneur whom, without loss of generality, we refer to as entrepreneur \( i \). All entrepreneurs own the same technology as in (1). The opportunity cost of lending is the risk-free interest rate \( r_f \), which is the per unit return from the money market, is normalized to zero. For the analysis of this section we assume that \( \alpha > 2\sqrt{2} - 1 \). This guarantees that each bank \( i \) has an expected per unit return \((1 - \hat{\theta}_i)\rho_i > 1 \) in equilibrium, and hence the bank will invest all its deposits in loans, and not in the money market. At date 3, when the loan obligations are paid off, the bank pays back \( r_i \) to the depositor. Both the borrowers and banks are protected by limited liability, i.e., if the project of the borrower of bank \( i \) fails neither the bank nor the depositor is repaid.

Further, the entrepreneurs, without loss of generality, are assumed not to have direct access to money market. Given the constant returns to scale production technology thus each entrepreneur \( i \) will invest the entire loan obtain in the project, i.e., \( I_i = D_i \). The returns of the risky projects are assumed to be independent across borrowers.

3.1 The optimal loan contracts

An optimal loan contract \( \rho_i^* \) and risk level \( \theta_i^* \) for the bank-borrower relationship \( i \) solve the following maximization problem:

\[
\max_{(\theta_i, \rho_i)} (1 - \theta_i)\rho_i D_i - D_i \tag{M'_i}
\]

subject to

\[
\theta_i = \arg\max_{\hat{\theta}_i} (1 - \hat{\theta}_i)(\alpha + \hat{\theta}_i - \rho_i)D_i = \frac{1}{2}(1 - \alpha + \rho_i) \equiv \theta(\rho_i), \tag{ICE_i}
\]

\[
(1 - \theta_i)(\alpha + \theta_i - \rho_i)D_i \geq 0 \tag{IRE_i}
\]

A bank cannot observe the choice of risk by its borrower, and hence limited liability implies a moral hazard problem in risk taking. Borrower \( i \) will choose the risk level in order to maximize her expected
utility which is asserted by the \textit{incentive compatibility} constraint (ICE). The constraint (IRE) is the \textit{individual rationality} constraint of entrepreneur \(i\) whose outside option is normalized to zero. The above maximization problem yields the optimal loan rate and risk level which are described in the following lemma.

\textbf{Lemma 1} \textit{The optimal loan rates charged to and risk shifting chosen by the entrepreneurs are given by:}

\[
\rho_i^* = \rho^* = \frac{1 + \alpha}{2} \quad \text{for all } i = 1, \ldots, n, \\
\theta_i^* = \theta^* = \frac{3 - \alpha}{4} \quad \text{for all } i = 1, \ldots, n.
\]

Since the banks and entrepreneurs are ex-ante identical, and the project returns are independent across borrowers, the optimal loan rates and risk levels are constant across entrepreneurs. Note also that when \(\alpha\), the per unit return to a safe project increases, the entrepreneurs tend to undertake less risk. Although both the bank and the borrower are risk neutral, limited liability of the entrepreneur induces the bank behave as risk averse and the entrepreneur as risk lover. If \(\alpha\) increases, the entrepreneur is required a lower margin in order to be incentivized to take less risk, and hence the loan rate increases.

\subsection*{3.2 The optimal deposit contracts}

At date 1, each bank \(i\) solves the following maximization problem:

\[
\max_{r_i} (1 - \theta^*)(\rho^* - r_i)D_i(r_i, r),
\]

where \(r\) is the deposit rate announced by the rival banks in a symmetric SPNE. It follows from Lemma 1 and the above maximization problem that

\textbf{Proposition 2} \textit{The equilibrium risk shifting is independent of the degree of deposit market competition. Moreover, the equilibrium deposit rate is monotonically increasing in the degree of deposit market competition, measured either by the degree of substitutability \(\delta\) or by the number of banks \(n\).

When the borrowers are the one who choose their investment strategies, it is obvious that this choice would depend only on the attributes of their own projects. Since the project returns are independent across borrowers, the terms of the loan contracts do not depend even on how many banks are there in the economy. Hence, neither the degree of substitutability in the deposit market nor the number of banks affects the equilibrium risk level of the economy. The intuition behind the fact that the equilibrium deposit rate is monotonically increasing in the degree of deposit market competition is similar to Proposition 1.\footnote{Note that the only equilibrium variable that depends on the deposit market competition is the symmetric deposit rate. Also, it is easy to show that the intermediation margin \(n(\delta, n) = (1/2)(1 + \alpha) - \tilde{r}(\delta, n)\) goes to zero as either \(\delta \to 1\) or \(n \to \infty\).}
4 Concluding remarks

In this very simple model, the terms of the loan contracts, and consequently the optimal level of risk chosen by each entrepreneur are independent across bank-borrower relationships since the project returns are independent. In general, lenders are able to write exclusive loan contracts with their borrowers which refrains borrowers from taking multiple loans. Therefore, the assumption of independence of the project returns is not so unnatural. The model could have been easily extended to a situation where each bank $i$ lends to an exclusive set of borrowers out of its total deposits $D_i$, which would yield similar results. The main objective of introducing loan contracts is to show that the two markets are in practice dichotomous. Had the project returns been correlated, the optimal risk shifting may have depended on the number of banks as in Martinez-Miera and Repullo (2010). But even in this case it would be difficult to conclude that equilibrium risk shifting depends on the degree of competition in the deposit market.

A more general model of loan market could have been rigged up where banks compete also in differentiated loan services. Consider a linear demand for loans with the degree of substitutability of loan services $\gamma \neq \delta$, which measures the degree of competition in the loan market. In this case, following Dam et al. (2014) it is easy to show that the equilibrium risk is decreasing in $\gamma$ and $n$, which conforms partially to the findings of Boyd and De Nicoló (2005). But, even in this case, the risk level would be independent of the degree of substitutability of the deposit market. Thus, a prudential regulatory policy to curb risk of bank failure would call for intervention in the loan market.

Appendix

Proof of Proposition 1

Consider the maximization problem $(M_i)$ of bank $i$. Substituting for $\Theta = \theta(r_i)$ into the bank’s objective function, its expected profit at $t = 0$ reduces to:

$$\Pi_i(r_i, r) \equiv [1 - \theta(r_i)][(\alpha + \theta(r_i) - r_i]D_i(r_i, r) = \left(\frac{1 + \alpha - r_i}{2}\right)^2 D_i(r_i, r),$$

where $r$ is the deposit rate offered by each of the rival banks in a symmetric equilibrium. Therefore, each bank $i$ chooses $r_i$ to maximize $\Pi_i(r_i, r)$. The first-order condition of the above maximization problem yields the best reply function $r_i(r)$ of bank $i$ which is given by:

$$\frac{1}{2}[1 + \alpha - r_i(r)][1 + \delta(n-2)] = -(1 - \delta) + [1 + \delta(n-2)]r_i(r) - \delta(n-1)r.$$

Thus, putting $r_i = r = r(\delta, n)$, we get

$$r(\delta, n) = \frac{2(1 - \delta) + (1 + \alpha)[1 + \delta(n-2)]}{2(1 - \delta) + [1 + \delta(n-2)]}. \quad (2)$$

The optimal risk level is thus given by:

$$\theta(\delta, n) = \frac{1 - \alpha + r(\delta, n)}{2} = \frac{(2 - \alpha)(1 - \delta) + [1 + \delta(n-2)]}{2(1 - \delta) + [1 + \delta(n-2)]}. \quad (3)$$
Differentiating (2) with respect to \( \delta \) we get
\[
\frac{\partial r(\delta, n)}{\partial \delta} = \frac{2(r(\delta, n) - 1) + (n - 2)(1 + \alpha - r(\delta, n))}{3(1 - \delta) + (n - 1)\delta}
\]
The above expression is strictly positive since \( \delta \in (0, 1) \) and \( n \geq 2 \). Since \( \theta(\delta, n) = \theta(r(\delta, n)) \), we have
\[
\frac{\partial \theta(\delta, n)}{\partial \delta} = \theta'(r(\delta, n)) \frac{\partial r(\delta, n)}{\partial \delta} = \frac{1}{2} \frac{\partial r(\delta, n)}{\partial \delta} > 0.
\]
To prove part (b) of the proposition differentiate (2) with respect to \( n \). This yields
\[
\frac{\partial r(\delta, n)}{\partial n} = \delta [1 + \alpha - r(\delta, n)] \frac{1}{3(1 - \delta) + (n - 1)\delta}.
\] (4)
Note that (2) can be written as
\[
r(\delta, n) = 1 + \frac{\alpha}{2(1 - \delta) + 1 + \delta (n - 2)} < 1 + \alpha.
\] (5)
Therefore, the expression in (4) is strictly positive. Now, similar to part (a) we have
\[
\frac{\partial \theta(\delta, n)}{\partial \delta} = \frac{1}{2} \frac{\partial r(\delta, n)}{\partial \delta} > 0,
\]
which completes the proof of part (b).

**Proof of Lemma 1**

Note that substituting for \( \theta_i \) from (ICE\(_i\)) into the expression of the borrower’s expected utility, her individual rationality constraint (IRE\(_i\)) reduces to:
\[
\left( \frac{1 + \alpha - \rho_i}{2} \right)^2 D_i \geq 0,
\]
which is always satisfied, and hence can be ignored. Using (ICE\(_i\)), bank \( i \)'s maximization problem reduces to:
\[
\max_{\rho_i} \left( \frac{1 + \alpha - \rho_i}{2} \right) \rho_i D_i - D_i.
\]
The first order condition of the above maximization problem yields
\[
\rho_i^* = \frac{1 + \alpha}{2}.
\]
Therefore, from (ICE\(_i\)) it follows that
\[
\theta_i^* = \theta(\rho_i^*) = \frac{3 - \alpha}{4}.
\]
Proof of Proposition 2

The first part follows trivially from Lemma 1. Now substituting for $\rho^*$ and $\theta^*$, the maximization problem of bank $i$ in the deposit market reduces to:

$$\max_{r_i} \left( \frac{1 + \alpha}{4} \right) \left( \frac{1 + \alpha}{2} - r_i \right) D_i(r_i, r).$$

Calculations similar to those in Proposition 1 yields

$$\tilde{r}(\delta, n) = \frac{2(1 - \delta) + (1 + \alpha)[1 + (n - 2)\delta]}{2[2 + (n - 3)\delta]}$$

in the symmetric SPNE. Differentiating the $\tilde{r}(\delta, n)$ with respect to $\delta$ and $n$, respectively we get

$$\frac{\partial \tilde{r}(\delta, n)}{\partial \delta} = \frac{\delta[1 + \alpha - 2\tilde{r}(\delta, n)]}{2[2 + (n - 3)\delta]}.$$

It is easy to show that $\alpha > 1$ implies $1 + \alpha - 2\tilde{r}(\delta, n) > 0$, and hence the above two expressions are strictly positive. This completes the proof of the proposition.

References


